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OF NOTED MATHEMATICIANS, ETC.

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BIOGRAPHY.

PROF. E. B. SEITZ, M. L. M. S.

BY B. F. FINKEL.

Professor Enoch Beery Seitz, the most distinguished mathematician of his day, was born in Fairfield county, Ohio, August 24, 1846, and died at Kirksville, Missouri, October 8, 1883. His father, Daniel Seitz, was born in Rockingham county, Virginia, December 17, 1791, and was twice married. His first wife's maiden name was Elizabeth Hite, of Fairfield county, Ohio, by whom he had eleven children. His second wife's maiden name was Catherine Beery, born in the same county, April 11, 1808, whom he married April 15, 1832, and by whom he was blessed with four sons and three daughters. He died near Lancaster, Ohio, October 14, 1864, in his seventy-third year, having been a resident of Fairfield county for sixty-three years.

Prof. Seitz, the third son by his father's second marriage, passed his boyhood on a farm, and like most men that have become noted, had only the advantages of a common school education. Possessing, however, a great thirst for learning, he applied himself to his books in private, and became a very fine scholar in the English branches, especially excelling in arithmetic. In the common school, though yet a little boy, he had greater power of arithmetical analysis than some of his teachers. He completed algebra at the age of fifteen without an instructor. He chose teaching as his profession, which he followed for a number of years with gratifying success.

He took a mathematical course in the Ohio Wesleyan University in 1870, but did not finish it or graduate. In 1872, he was elected one of the teachers in the Greenville High School, which position he held till 1879. On the 24th of June, 1875, he married Miss Anna E. Kerlin, one of Dark county's most refined ladies. In 1879, he was elected to the chair of mathematics in the Missouri State Normal school, Kirksville, Missouri, which position he held till death called him from the confines of earth, ere his star of fame had reached the zenith of its glory. He was striken by that "demon of death," typhoid fever, and passed the mysterious shades, to be numbered with the silent majority, on the 8th of October, 1883. On March the 11th, 1880, he was elected a member of the London Mathematical Society, being the fifth American so honored.

Prof. Seitz was in mathematics what Demosthenes was in oratory; Shakespeare in poetry; and Napoleon in war: the equal of the best, the peer of all the rest.

He began his mathematical course in 1872 by contributing solutions to the problems proposed in the "Stairway" department of the *Schoolday Magazine*, conducted by Artemas Martin. His masterly and original solutions to difficult Average and Probability problems, soon attracted universal attention among mathematicians.

Dr. Martin, being desirous to know what works he had treating on that difficult subject, was greatly surprised to learn that he had no works upon the subject, but had learned what he knew about that difficult department of mathematical science by studying the problems and solutions in the *Schoolday Magazine*. He then contributed to the *Analyst*, the *Mathematical Visitor*, the *Mathematical Magazine*, the *School Visitor*, and the *Educational Times*, of London, England.

In each of these journals, Prof. Seitz was second to none, as his logical and

Prof. E. B. Seitz.

classic solutions to Average and Probability problems, rising as so many monuments to his untiring patience and indomitable energy and perseverance will attest.

"His name first appears as a contributor to the *Educational Times* in Vol. XVII., of the *Reprint*, year 1873. To Vol. XXI., he contributed a solution of the problem, "Find the average area of a spherical triangle." His solution takes up just twelve lines, and it was the only solution received by the editor.

In Vol. XXXII., p. 105, he solved the problem, "If A, B, C, and D are four points taken at random in the surface of a given circle; show that the chance that E, the intersection of the straight lines through A, B and C, D, lies between A,B and between C,D, is $\frac{1}{3}-\frac{35}{36\pi^2}$".

In this communication his genius is displayed to a grand advantage; he is at home in his favorite field of investigation. The answer requires the evaluation of an octuple integral. The work is ably done and furnishes a fine specimen of what a classic solution ought to be. To Vol. XXXIX., he contributed a number of problems and solutions, and three solutions of problems proposed by other contributors; three have his own solution appended—no others, apparently, having been received by the editor. They look at first sight like a forest of definite integral symbols, but they are evidently in the line of his favorite pursuit, "Average and Probability." The *Mathematical Visitor* is adorned with some of his choicest solutions, which ever display his mathematical genius. His first solution for the *Mathematical Visitor* is of the following problem: "Find the equation to the locus of the centers of all the circles that can be incribed in a given semi-ellipse." The solution undoubtedly required a vast amount of patience and profound insight into the intricate and subtle relations which had to be traced in order to reach a result. The copying of the answer would exhaust the patience of the average student. On page 33, is a solution of the following problem: "A straight tree growing vertically on the side of a mountain was broken off by the wind, but not severed; find the chance that the top reaches the ground."

On page 37, is a solution to a prize problem. "A boy stepped upon a horizontal turn-table while it was in motion, and walked across it keeping all the time in the same vertical plane. The boy's velocity is supposed to be uniform in his track on the table, and the motion of the table toward him. The velocity of a point in the circumference of the turn-table is n times the volocity of the boy along the curve he describes. Required the nature of the curve the boy describes on the table, and the distance he walks while crossing it (1) when n is less than 1, (2) when n equals 1, (3) when n is greater than 1." His solution is a pantheon in grandeur and sublimity, adorned with the richest ornament of thought. Though the calculus acting as a radius vector, sweeping from one limit to another and embracing every element between the limits of that to which it is applied, yet through what a labyrinth of complex and hidden principles is the mind obliged to pass in order to see that the adjustment of this instrument will contain every element in its flight from one limit to another.

This solution alone would have been sufficient to place his name high in the category of American mathematicians. But this masterly solution is only one of the magnificent edifices of thought he erected for the children of men, in which they may congregate and learn something of the vastness and everlasting grandeur of its construction. On page 58 of the *Mathematical Visitor*, his name is attached to the solution of another "prize problem":

If $y=x+ax^2+bx^3+cx^4+\&c.$ ......(1), and,

$x = y + Ay^2 + By^3 + Cy^4 +$ &c. ....(2) ; find, A, B, C, D, E, F, G, H, I, J, K, L, M, N, P, in terms of a, b, c, d, e, f, g, h, i, j, k, l, m, n, p.

The principle involved in this problem is not the difficulty in effecting a solution; it is the prodigious amount of labor required in order to obtain a correct result. Any one acquainted with the process of reverting a series, knows that the work is tedious. But this wonderful mathematician, for whom no problem was too abstruse or labor too great, accomplished the work. His value of P covers about half of a quarto page. He finds the values of A, B, C, ... I, by actually performing the work, when the acuteness of his intellectual vision discerned a law by which the succeeding coefficients could be easily written out.

On page 79, is a solution to the problem: "Find the average distance between two points taken at random within a rectangular solid, edge, a, b, c." The solution, with a beautiful figure, covers an entire page, and is grand and imposing. On page 157, appears a fine solution to the problem: "If three dice be piled up at random on a horizontal plane, what is the probability that the pile will not fall down?"

On page 21, Vol. II, he has given a solution to the problem: "A cube is thrown into the air and a random shot is fired through it; find the chance that the shot passes through the opposite faces."

This problem had been proposed in 1864, by the great English mathematician, Prof. Woolhouse, who solved it with great labor. It was said by an eminent mathematician of that time, that the task of writing out a copy of that solution was worth more than the book in which it was published.

No other mathematician seemed to have the courage to investigate this problem after Prof. Woolhouse gave his solution to the world, till Prof. Seitz took it up and demonstrated it so elegantly in half a page of ordinary type, that he fairly astonished both the mathematicians of Europe and America.

Prof. Woolhouse was the best English authority on probabilities, even before Prof. Seitz was born.

It was the solution of this problem that won for Prof. Seitz the acknowledgment of his superior ability, in this abstruse department of mathematics, over any other man in either hemisphere. These are only a few of the many problems to which he has furnished the finest solutions. In studying his work, one is struck with the simplicity to which he has reduced the solutions of some of the most intricate problems. When he grasped a problem in its entirety, he had mastered all problems of that class. He would so vary the conditions in thinking of one special problem and in effecting a solution that he had generalized all similar cases, so exhaustive was his analyses. Behind the words he saw all the ideas represented. These he translated into symbols, and then he handled the symbols with a facility that has never been surpassed.

What he might have accomplished in his maturer years, no man may say; but at the age of thirty-seven he laid down his pen, and gave to God, from whence it came, the casement and the key of his mighty intellect, leaving his impress indelibly stamped upon the thinking and scientific world for all time. He has written his name in characters of gold and prismatic hues on the pinnacle of the temple of fame, and his good work will ever be cherished in the memories of those whom he has left behind.

He was a man of the most singularly blameless life; his disposition was amiable; his manner gentle and unobtrusive; and his decision, when circumstances demanded it, was prompt and firm as the rocks.

He did nothing from impulse; he carefully considered his course; and with a wise judgment came to conclusions that his conscience approved, and when his de-

cision was made, it was unalterable. He never made an open profession of religion, yet he was an intensely religious man. He rested his hopes on the sacrifice of the tender and loving Saviour, and we feel satisfied that he has entered into that rest which remaineth for the people of God.

Professor Seitz was not only a mathematician, but he was eminently proficient in other branches of knowledge. His mind was cast in a gigantic mold, "Being devout in heart as well as great in intellect, 'signs and quantities were to him but symbols of God's eternal truth' and he 'looked up through nature up to natures God.' Professor Seitz, in the very appropriate words of Dr. Peabody, regarding Benjamin Peirce, Professor of Mathematics and Astronomy in Harvard University, 'saw things precisely as they are seen by the infinite mind. He held the scales and compasses with which the eternal wisdom built the earth and meted out the heavens. As a mathematician, he was adored with awe. As a man, he was a christian in the whole aim and tenor of life.' " No mathematician was so universally loved and honored by his contemporaries as was Professor Seitz.

He did not gain his knowledge from books, for his library consisted of only a few books and periodicals. He gained such a profound insight into the subtle relation of numbers by close application with which he was particularly gifted. He was not a mathematical genius, that is, as usually understood, one who is born with mathematical powers fully developed; but he was a genius in that he was especially gifted with the power to concentrate his mind upon any subject he wished to investigate. This happy faculty of concentrating all his powers of mind upon one topic to the exclusion of all others and viewing that topic from all sides, enabled him to proceed with certainty. Thread by thread and step by step, he took up,and followed out long lines of thought and arrived at correct conclusions. The darker and more subtle the question appeared to the average mind, the more eagerly he investigated it. No conditions were so complicated as to discourage him.

He left a wife and four sons, one of whom has gone to join his father in ths realms of eternal peace. His mother, now (1894) eighty-six years old is still living and enjoying good health.

LOWEST INTEGERS REPRESENTING SIDES OF A RIGHT TRIANGLE.

By LEONARD E. DICKSON, B. Sc., Fellow in Pure Mathematics, University of Texas.

Let the whole numbers expressing the lengths of the sides of a right-angled triangle be reduced to their lowest forms by dividing out their highest common divisor.

Call the resulting numbers a, b, and c.

1. They can not all be *even* numbers. For if so, they would still have the common divisor 2.

2. They can not all be *odd* numbers. For $a^2+b^2=c^2$; and, if a and b are odd, there squares are odd, and the sum of their squares even. But c^2 being even, c must be even.

3. c, the greatest of the three, must be *odd*; and, of the remaining two, one must be even and the other odd.

$b^2=c^2-a^2=(c+a)(c-a)$. Suppose b is even. The product of $(c+a)$ and $(c-a)$ is even;hence,both factors are even. [For the case in which one factor is even and the other odd is impossible, since the sum and the difference of two numbers are both even or both odd.] Hence, since the sum and difference of a and c are both even, they are themselves both even or both odd. But by 1, a and c can not both be even if b is even. Hence, when b is even, a and c are both odd.

Similarly, $a^2=c^2-b^2=(c-b)(c+b)$. Reasoning exactly as before, we find that when a is even, b and c are both odd. Hence,we may conclude that c is always odd; and that b is even when a is odd, but odd when a is even.

4. As a corollary to 3, we get the theorem: When the sides of a right triangle are expressed by integers, whether reduced by dividing out their common divisor or not so reduced, the perimeter of the triangle is expressed by an *even* number.

Formulae for obtaining values for a, b, and c.

5. *Rule of Pythagoras:* n, $\frac{n^2-1}{2}$, and $\frac{n^2+1}{2}$, when n is *odd*. The number $\frac{n^2-1}{2}$ is always divisible by 4. For, writing $(2k+1)$ for the odd number n, we get $\frac{n^2-1}{2}=\frac{4k^2+4k}{2}=2k(k+1)$, where $k(k+1)$ is even whether k be even or odd. As a corollary, the area of the triangle formed is expressed by an *even* number.

6. *Plato's Rule:* m, $\frac{m^2}{4}-1$, $\frac{m^2}{4}+1$, where m is *even*. THEOREM: m is always divisible by 4. For $\frac{m^2}{4}+1$ must [by 3.] be odd. Thus $\frac{m^2}{4}$ is even. Hence, m^2, being divisible by 8, must be divisible by 16, in order to be a perfect square. As before, the area of the triangle formed is expressed by an *even* number.

7. *Euclid's Rule:* $\sqrt{xy}$, $\frac{x-y}{2}$, $\frac{x+y}{2}$, where x and y are both even or both odd, and where xy is a perfect square. Further, x and y must have no common factor greater than 2; for if so, the sides a, b, and c would contain this factor.

8. *Rule of Maseres:* m, $\frac{m^2-n^2}{2n}$, $\frac{m^2+n^2}{2n}$. Since the last number is an integer, (m^2+n^2) is even and, hence, m and n are both even or both odd. m being one number, n is evidently the difference of the other two. [The numbers found by Maseres' Rule are usually expressed as (m^2-n^2), $2mn$, and (m^2+n^2), derived from those above by multiplying through by $2n$. As they thus contain a common factor, they are replaced by the ones given above, which are in their lowest form.]

Correlation of the Four Rules.

9. The Rule of Pythagoras and Plato's Rule are only special cases of the Rule of Maseres; while Euclid's Rule is that of Maseres under a different form. For let $n=1$ in the numbers found by the Rule of Maseres, viz.; m, $\frac{m^2-n^2}{2n}$, and $\frac{m^2+n^2}{2n}$, where m and n are both even or both odd, and we get the numbers given by the Rule of Pythagoras, viz.; m, $\frac{m^2-1}{2}$, $\frac{m^2+1}{2}$, where m is *odd*. Let $n=2$, and we get the numbers given by Plato's Rule: m, $\frac{m^2}{4}-1$, and $\frac{m^2}{4}+1$, where m is *even*. Let $n=y$ and $\frac{m^2}{n}=x$, and we get the numbers given by Euclid's Rule; $\sqrt{xy}$, $\frac{x-y}{2}$, $\frac{x+y}{2}$, where x and y are both even or both odd, and where xy is a perfect square (m^2).

A New Rule.

10. Let a, b, and c be the numbers reduced to their lowest form which represent the sides of a right-angled triangle. Let $(c-a)=m$; $(c-b)=n$, where m and n are integers, one being even and the other odd.

$$\therefore\ (c-m)^2+(c-n)^2=c^2,$$
$$c^2-2c(m+n)+m^2+n^2=o,$$
$$c^2-2c(m+n)+(m+n)^2=2mn,$$
$$c-(m+n)=\pm\sqrt{2mn},$$
$$c=m+n\pm\sqrt{2mn}.$$

Since $2c=(a+b)+(m+n)$ and $c<(a+b)$; $\therefore\ c>(m+n)$. Hence, we must take the plus sign before the radical.

$$\therefore\ c=m+n+\sqrt{2mn};$$
$$a=m+\sqrt{2mn};\ \text{and}$$
$$b=n+\sqrt{2mn}.$$

By inspection, m and n must have no common factor; for this factor would appear in a, b, and c. My rule may be stated formally, thus:

Take any two integers, m and n, such that one is even and the other odd, and such that their product is a perfect square; then the three sides will be $m+\sqrt{2mn}$, $n+\sqrt{2mn}$, $m+n+\sqrt{2mn}$.

By Euclid's Rule (to which the other three are special cases or equivalent), we take any two integers, m and n, such that both are even or both odd, and such that their product is a perfect square; then the sides are $\sqrt{mn}$, $\frac{m-n}{2}$, $\frac{m+n}{2}$.

Complete Table of Values.

11. We may get an absolutely complete table of values for a, b, and c, and one in which each set is reduced to its lowest form, by employing either Euclid's Rule or my rule, in each case observing the necessary conditions and restrictions.

In Euclid's set of values, $\sqrt{xy}$, $\frac{x-y}{2}$, and $\frac{x+y}{2}$, x and y must both be even or both odd; and further they can have no common divisor higher than 2, since a, b, and c would then have this divisor. The *practical* rule, which gives every possible set of values in their lowest form, and which gives each set but once is as follows: Let y be any *odd* square number whatever, and x any greater *odd* square number not divisible by y. Thus if $y=9$, x may be 25, 49, 121,169, etc.

In my set of values, $m+\sqrt{2mn}$, $n+\sqrt{2mn}$, $m+n+\sqrt{2mn}$, m and n must one be even and the other odd; and further, they can have no common divisor whatever, since a, b, and c would then have this divisor. My *practical* rule, which gives every possible set of values in their lowest form, and which gives this set but once, is this: Let m be any *odd* square number whatever, and let n be the *double* of *any* square number whatever not divisible by m. Thus if $m=9$, n may be the *double* of 1, 4, 16, 25, 49, 64, 100, etc.

Additional Theorems.

12. One out of each set of values found by my practical rule is divisible by 4. Let $m=(2l+1)^2$ and $n=2k^2$. Then $\sqrt{2mn}=2k(2l+1)$. Thus $n+\sqrt{2mn}$ $=2k^2+2k(2l+1)=2k(2l+2)=4k(l+1)$.

13. One out of each set of numbers found by the practical rule derived from Euclid's Rule is divisible by 4. Let $y=(2l+1)^2$ and $x=(2k+2)^2$. $\frac{x-y}{2}=$ $2(l^2-k^2+l-k)=2(l-k)(l+k+1)$. Now since $(l-k)$ and $(l+k)$ are both even or both odd, $(l-k)$ and $(l+k+1)$ are one even and the other odd. $\therefore \frac{x-y}{2}$ is divisible by 4.

14. As a corollary, the area of *any* right-angled triangle whose sides are integers is express by an *even* integer.

15. The radii of the inscribed and three escribed circles of any right-triangle whose sides are integers are all expressed by integers. The trigonometric expressions for r, r_a, r_b, r_c, viz.; $\frac{\triangle}{s}, \frac{\triangle}{s-a}, \frac{\triangle}{s-b}, \frac{\triangle}{s-c}$, become for the right-triangle $(s-c)$, $(s-b)$, $(s-a)$, and s respectively, where s is the semi-perimeter(always an integer). Hence, the radii are all integers.

Dr. Halsted's *Mensuration* contains a three-page table of values for a, b, and c. It is absolutely *complete* as far as the 59th set; but beyond this, sets are occasionally omitted.

SCALENE TRIANGLES.

Let a, b, and c be three integers having no common divisor which represent the sides of a scalene triangle whose *area* may be expressed by an *integer*.

1. They can not all be even.
2. They can not all be odd.

For $\triangle=\sqrt{s(s-a)(s-b)(s-c)}=\frac{1}{4}\sqrt{2(a^2b^2+a^2c^2+b^2c^2)-(a^4+b^4+c^4)}$. Now

if a, b, and c all be odd, $(a^4+b^4+c^4)$ is odd and the expression under the radical sign must be odd. Thus the area could not be expressed by an integer.

3. No two of them can be even. Proved as in (2).

4. Hence, of the three, one is even and the other two odd.

5. As a corollary, the perimeter of the triangle will be expressed by an *even* number.

6. THEOREM: The area of the triangle is expressed by an *even* number. Let $a=2l+1$; $b=2m+1$; $c=2n$, as one of the sides is even and the other two odd. $\therefore\ \frac{a+b+c}{2}=l+m+n+1$; $\frac{a+b-c}{2}=l+m-n+1$; $\frac{a-b+c}{2}=l-m+n$; $\frac{-a+b+c}{2}=-l+m+n$. Hence, of these four quantities, two are even and two odd. $\therefore$ area, $=\sqrt{\frac{a+b+c}{2}\cdot\frac{a+b-c}{2}\cdot\frac{a-b+c}{2}\cdot\frac{-a+b+c}{2}}$, is divisible by 2.

7. Problem: To find three integers a, b, and c representing the sides of a triangle whose *area* may be expressed by an *integer*.

$$\triangle=\tfrac{1}{4}\sqrt{2(a^2b^2+a^2c^2+b^2c^2)-(a^4+b^4+c^4)}.$$

Solving for a^2, we get $a^2=b^2+c^2\pm2\sqrt{b^2c^2-4\triangle^2}$.

Similarly, $b^2=a^2+c^2\pm2\sqrt{a^2c^2-4\triangle^2}$.

Similarly, $c^2=a^2+b^2\pm2\sqrt{a^2b^2-4\triangle^2}$.

Let a be greater than either b or c. Then $\angle A$ may be acute or obtuse; while $\angle B$ and $\angle C$ are both acute. Hence, writing l for $-\sqrt{b^2c^2-4\triangle^2}$, m for $-\sqrt{a^2c^2-4\triangle^2}$, n for $-\sqrt{a^2b^2-4\triangle^2}$, l may be $\pm$, while m and n *must* be $+$. Further, the conditions of the problem require that l, m, and n be integers. We have $a^2-b^2-c^2=\mp2l$; $b^2-a^2-c^2=-2m$; $c^2-a^2-b^2=-2m$. $\therefore\ a^2=m+n$; $b^2=n\pm l$; $c^2=m\pm l$. Thus the conditions that $b^2c^2-4\triangle^2$, $a^2c^2-4\triangle^2$, $a^2b^2-4\triangle^2$, be perfect squares, l^2, m^2, and n^2, respectively, reduce to the single condition that $4\triangle^2=mn\pm ml\pm nl$. Hence, the Rule to find a, b, c such that area $\triangle ABC$ be integral is: Take any three integers $\pm l$, m, and n, such that one is odd and the other two are even, and such that the sums of them two at a time are perfect squares, and lastly such that the sum of their products two at a time is a perfect square; then $a=\sqrt{m+n}$; $b=\sqrt{m\pm l}$; $c=\sqrt{m\pm l}$; and $\triangle=\frac{1}{2}\sqrt{mn\pm ml\pm nl}$.

8. Let fall a $\perp$ from each vertex of the triangle upon the opposite side. We can express the intercepts on the sides either in terms of $l,m,$and n or of $a,b,$and c. For example, those on c are $\frac{\pm l}{\sqrt{m\pm l}}$ and $\frac{m}{\sqrt{m\pm l}}$ or in terms of a, b, c, $\frac{b^2+c^2-a^2}{2c}$ and $\frac{a^2+c^2-b^2}{2c}$. Thus in general these intercepts are not integers.

9. If two right-triangles, having a perpendicular side of the one equal to a perpendicular side of the other, be placed so that these sides coincide, the other perpendicular sides falling in the same straight, then one of the two possible scalene triangles are formed. The sides and the area of these scalene tri-

angles are expressed by integers, providing the three sides of each triangle be integers. But from (8) we find that the sides of the new triangles may have a common factor. Guarding this point, we may form from a table of the lowest integers representing sides of a right triangle a corresponding table for a scalene triangle whose *area is an integer*. A four-page table of this kind is given in Dr. Halsted's *Mensuration*.

POSTULATE I. OF EUCLID'S ELEMENTS.

By Professor JOHN L. LYLE, Ph, D., Westminister College, Fulton, Missouri.

"Let it be granted that a straight line may be drawn from any one point to any other point." Euclid lays down the statement just quoted as his first postulate regulative of geometrical constructions. Wherever any two points may be located in unbounded space, Euclid assumes that a straight line may be drawn from one of them to the other.

Such a line is finite in length, of course, according to the definition that "a *finite* straight line is one that has *two ends*".

The assumption that a straight line of *infinite* (boundless) length can be drawn between two points in space is not only anti-Euclidean but also destructive of the logical law of non-contradiction which forbids that contradictory marks shall be attributed to any straight line whatever. A line can not have two ends and at the same time be without ends,(infinite,that is,unbounded).

In every rectilineal triangle, there are three angular points and each of the three sides may be constructed in strict harmony with the 1st postulate of Euclid. Each of these sides has the distinctive marks of a finite straight line, to wit: two ends.

Euclid proves in proposition XVII, Book I., that "Any two angles of a triangle are together less than two right angles". This proposition is the converse of Euclid's 12th axiom about which so much has been written.

John Playfair states that axiom as follows: "If a straight line meet two straight lines, so as to make the interior angles on the same side of it less than two right angles, these straight lines being continually produced will at length meet on the side on which the angles are less than two right angles."

Lobatschewsky, in his theorem 19,demonstrates the proposition that the angle-sum of a rectilineal triangle can not be greater than two right angles. Then assuming the falsity of Euclid's 12th axiom, he concludes (whether logically or illogically) that the angle-sum of a rectilineal triangle is less than two right angles.

If now, using Fig. 1 in a new way, (c) be taken as the product of (a) and (b) by multiplication of ordinates, we readily obtain, by a similar method of proof, the result,

$$\frac{dy}{dx}=u\frac{dv}{dx}+v\frac{du}{dx}.$$

This result is expressed to the eye by a figure similar to Fig. 4, wherein the number of linear spaces in Tt=(the number of linear spaces in CM)×(the number in $T't'$)+(number in $T''t''$)×(the number in AM).

Thus when the slopes of two curves are known, that of their product can be at once written down.

In this way all the ordinary formulas of differentiation are obtained as applied to curves.

The student should now be introduced to the idea of function, that is of quantity independent of any spacial relations, and brought to see the greater freedom and flexibility of treatment obtained by its use.

(TO BE CONTINUED.)

ARITHMETIC.

Conducted by B. F. FINKEL, Kidder, Missouri. All contributions to this department should be sent to him.

SOLUTIONS TO PROBLEMS.

1. I. Reduce $\frac{1}{3}$ to 8ths.

1st Solution.

II. 1. $1=8$ eighths.
2. $\frac{1}{3}=\frac{1}{3}$ of 8 eighths$=2\frac{2}{3}$ eighths.

III. ∴ $\frac{1}{3}=2\frac{2}{3}$ eighths, or $\dfrac{2\frac{2}{3}}{8}$.

2nd Solution.

II. 1. $1=\frac{8}{8}$.
2. $\frac{1}{3}=\frac{1}{3}\times\frac{8}{8}=\dfrac{2\frac{2}{3}}{8}$.

III. ∴ $\frac{1}{3}=\dfrac{2\frac{2}{3}}{8}$.

2. I. Reduce $\frac{5}{6}$ to a fraction whose numerator is 11.

II. 1. $1=\frac{11}{11}$
2. $\frac{5}{6}=\frac{5}{6}\times\frac{11}{11}=\frac{11}{\frac{66}{5}}=\dfrac{11}{9\frac{1}{6}}$.

III. ∴ $\frac{5}{6}=\dfrac{11}{9\frac{1}{6}}$, a fraction whose numerator is 11.

3. Proposed by L. B. Hayward, Superintendent of Schools, Bingham, Ohio.

Bought an article and sold it for 3% less than it cost me; bought it back paying 3% more than I sold it for. I lost $12.00 by the transaction. What did the article cost at first?

Solution by the Proposer.

Let 100%=the cost at first.

Then 97%=selling price.

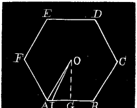
And 3% of .97=.02$\frac{91}{100}$.
97%+2$\frac{91}{100}$%=99$\frac{91}{100}$%=cost on repurchasing.
Loss is 100%−(97−2$\frac{91}{100}$)%=5$\frac{91}{100}$%.
∴ 5$\frac{91}{100}$%=\$12.00.
Then 1%=\$2.0305.
And 100%=\$203.05.

4. I. What is the number of acres in a field in the form of a regular hexagon, if it contains as many acres as there are boards in the fence inclosing it, the boards being $l=8\sqrt[4]{3}$ feet long and the fence $n=6$ boards high?

Solution by B. F. FINKEL, Professor of Mathematics in Kidder Institute, Kidder, Missouri.

Construction.— Let $ABCDEF$ be the field, O the center, AI the length of a panel of the fence. Connect A and I with the center of the field by the lines AO and IO. Draw OG perpendicular to the side AB. Then

II.
1. $AI=8\sqrt[4]{3}$ feet, and
2. *area* of $AOI=6A=261360$ sq.ft., since there are as many acres in the field as there are boards in the fence inclosing it and the panel being 6 boards high.
3. $\frac{1}{2}(AI\times OG)$=the area of the triangle $AOI=\frac{1}{2}(8\sqrt[4]{3}\times OG)=4\sqrt[4]{3}\times OG$.
4. $\therefore 4\sqrt[4]{3}\times OG=261360$, the number of square feet in the triangle AOI, whence
5. $OG=261360\div4\sqrt[4]{3}=\dfrac{65340}{\sqrt[4]{3}}$. But
6. $OG=\sqrt{[AO^2(=AB^2)-AG^2(=\frac{1}{2}AB)^2]}$ $=\frac{1}{2}AB\sqrt{3}$.
7. $\therefore\frac{1}{2}AB\sqrt{3}=\dfrac{65340}{\sqrt[4]{3}}$, whence
8. $AB=\dfrac{130680}{\sqrt[4]{3^3}}$ ft., the length of a side of the field. Then
9. $\dfrac{130680}{\sqrt[4]{3^3}}\div8\sqrt[4]{3}=5445$, the number of panels on a side, and
10. $6\,(6\times5445)=196020$, the number of acres in the field.



III. ∴ There are 196020 acres in the field.

Remark.— If we let l=the length of a rail and n, the number of rails in a panel, the number of acres in a hexagonal field will be $174240n^2\div l^2\sqrt{3}$. From the nature of the problem, n must be integral. ∴ The number of acres and the length of the rails can not both be rational. The above solution is also applicable when the field is in the form of a square.

PROBLEMS.

5. Proposed by E. E. KINNEY, Anaconda, Montana.

A board is 16 in. long and 9 in wide. How may it be cut in two parts that the parts joined together may form a square?

6. **Proposed by B. F. FINKEL, Professor of Mathematics in Kidder Institute, Kidder, Missouri.**

What is the volume of a regular pentagonal pyramid, each side of whose base is 10 feet and the altitude 20 feet?

7. If an article had cost me 10% less, the gain would have been 12% more; what was the gain per cent.? [Selected from *Brook's Higher Arithmetic.*]

8. **Proposed by EARL D. WEST, West Middleburg, Logan County, Ohio.**

The number of men in a side rank of a solid body of militia is to the number of men in the front rank as 2 is to 3; if the length and breadth be increased so as to number each 4 men more, the whole body will then contain 2320 men. How many men in the militia?

9. **Proposed by O. S. KIBLER, Superintendent of Schools, West Middleburg, Logan County, Ohio.**

Four logs of uniform thickness whose diameters are each 4 feet, lie side by side and touch each other. In the crevices of these logs lie three logs 3 feet in diameter, and in the crevices of the three logs lie two logs whose diameters are 2 feet. What must be the diameter of a log to lie on the top of the pile and touch the two logs and the middle one of the three logs?

10. **Proposed by MISS LECTA MILLER, B. L., Professor of Natural Science and Art, Kidder Institute, Kidder, Missouri.**

A carpenter is obliged to cut a board, that is in the form of a trapezoid, crosswise into two equivalent parts. The board is 12 ft. long, 2 ft. wide at one end, and one foot wide at the other. How far from the narrow end must he cut?

11. **Proposed by L. B. HAYWARD, Superintendent of Schools, Bingham, Ohio.**

What length of rope will be required to draw water from a well, it being 38 feet to the water, the sweep to be supported by an upright post 20 feet high, and standing 20 feet from the well, and the foot of the sweep to strike the ground 20 feet from the foot of the upright post?

ALGEBRA.

Conducted by J. M. COLAW, Monterey, Va. All contributions to this department should be sent to him.

SOLUTIONS TO PROBLEMS.

1. **Proposed by W. L. HARVEY, Portland, Maine.**

$(2x^2-1)^2-(2x^2-4x-1)(2x^2-1)=1$; find the value of x by quadratics.

Solution by the Proposer.

Performing the opperations indicated and collecting, the equation reduces to $8x^3-4x=1$. Multiplying both sides of this equation by $8x$ and adding $16x^2+1$ to both sides of the resulting equation, we have $64x^4-16x^2+1=16x^2+8x+1$, from which $8x^2-1=4x+1$. Then $8x^2-4x=2$, from which $x=\frac{1}{4}(1\pm\sqrt{5})$. Using the minus sign this will prove true.

2. **Proposed by Professor P. H. PHILBRICK, C. E., Lake Charles, Louisiana.**

Find x from the equation, $x^3+18x=1529$.

Solution by the Proposer.

Multiply by x, then $x^4+18x^2=1529x=139\times 11x$. Again, x^4+139x^2+

$\left(\frac{139}{2}\right)^2=121x^2+139\times 11x+\left(\frac{139}{2}\right)^2$. Extract square root and have,

$x^2+\frac{139}{2}=11x+\frac{139}{2}$ or $x^2=11x$.

PROBLEMS.

3. **Proposed by Professor H. A. WOOD, A. M., Hoboken, New Jersey.**

If $x^6-y^6=665$, and $x^3y+xy^3=78$, find x and y.

4. **Proposed by L. E. PRATT, Tecumseh, Nebraska.**

If Σm, Σm^3, Σm^5, ... Σm^{2n-1} are the sums of the 1st, 3rd, 5th,...... $(2n-1)$th powers of the first m natural numbers, prove that $n\Sigma m^{2n-1}+\frac{n(n-1)(n-2)}{\lfloor 3}\Sigma m^{2n-3}+\frac{n(n-1)(n-2)(n-3)(n-4)}{\lfloor 5}\Sigma m^{2n-5}+\ldots.=2^{n-1}\Sigma^n_m$.

5. **Proposed by WILLIAM E. MAY, Jonesboro, Tennessee.**

A, B, and C went to market, each with 10, 30, and 50 eggs, respectively. On their way to market, they agreed to sell their eggs at the same price per dozen so as to realize an equal integral number of cents. How much did they receive?

6. **Proposed by L. E. PRATT, Tecumseh, Nebraska.**

A vessel is to be filled with water by two pipes. The first pipe is kept open during m-nth of the time which the second would take to fill the vessel; then the first pipe is closed and the second is opened. If the two pipes had kept open together, the vessel would have been filled t hours sooner, and the first pipe would have brought in p-qth of the quantity of water which the second pipe really brought in. How long would it take each pipe alone to fill the vessel?

7. **Proposed by O. S. KIBLER, Superintendent of Schools, West Middleburg, Logan county, Ohio.**

A's age equals B's age plus the cube root of C's age; B's age equals C's age plus the cube root of A's age plus 14 years; and, C's age equals the cube root of A's age plus the square root of B's age. What is the age of each?

8. **Proposed by H. M. CASH, Salesville, Ohio.**

The longer side BC of a field in the form of a parallelogram is a (78) rods: the sum of its shorter side AB, and greater diagonal AC is b (114) rods; the distance from B at right angles with AB to a tree standing on AC, is c (32) rods. Find the area of the field, and the distance from the tree to the corners A, C, and D.

GEOMETRY.

Conducted by B. F. FINKEL, Kidder, Missouri. All contributions to this department should be sent to him.

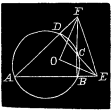
SOLUTIONS TO PROBLEMS.

1. **Proposed by B. F. FINKEL, Professor of Mathematics in Kidder Institute, Kidder, Missouri,**

Show that the bisectors of the angles formed by producing the sides of an inscribed quadrilateral intersect each other at right angles.

Solution by the Proposer.

Let $ABCD$ be the inscribed quadrilateral and FO and EO the bisectors of the angles F and E, respectively, formed by producing the sides of the quadrilateral. Denote the angle EAF by A; AFB, by F; BFE, by F'; AED, by E; DEF, by E'; $FCE,=DCB$, by C; and FOE, by O. Then $A+C=2$ rt. angles..(1); being opposite angles of an inscribed quadrilateral. Also, in the triangle AFE, $A+F+F'+E+E'=2$ rt. angles..(2); in the triangle FOE, $\frac{1}{2}F+F'+\frac{1}{2}E+E'+O=2$ rt. angles....(3); and, in the triangle FCE, $F'+E'+C=2$ rt. angles....(4).



Multiplying (3) by two and subtracting (4) from the resulting equation, we have $F+F'+E+E'+2O-C=2$ rt. angles..(5). Subtracting (5) from (2), we have $A+C-2O=0$, whence $2O=A+C=2$ rt. angles. $\therefore$ $O=$a rt. angle.

Q. E. D.

PROBLEMS.

2. Show that $\frac{1}{2}\pi=\left[\frac{2.\ 4.\ 6.\ 8.\ 10}{1.\ 3.\ 5.\ 7.\ 9}\cdots\right]^2$, *Wallis's* expression for π.

[Selected from *Bowser's Trigonometry.*]

3. If A be the area of the circle inscribed in a triangle, A_1, A_2, A_3 the areas of the escribed circles, show that $\frac{1}{\sqrt{A}}=\frac{1}{\sqrt{A_1}}+\frac{1}{\sqrt{A_2}}+\frac{1}{\sqrt{A_3}}$.

[Selected from *Todhunter's Plane Trigonometry.*]

4. Three circles whose radii are a, b, and c touch each other externally; prove that the tangents at the points of contact meet in a point whose distance from any one of them is $\left\{\frac{abc}{a+b+c}\right\}^{\frac{1}{2}}$. [Selected from *Todhunter's Plane Trigonometry.*]

5. **Proposed by ADOLPH BAILOFF, Durand, Wisconsin.**

If from a varible point in the base of an isosceles triangle, perpendiculars are drawn to the sides, the sum of the perpendicular is constant and equal to the perpendicular let fall from either extremity of the base to the opposite side.

6. **Proposed by EARL D. WEST, West Middleburg, Logan county, Ohio.**

Having given the sides 6, 4, 5, and 3 respectively of a trapezium, inscribed in a circle, to find the diameter of the circle.

7. **Proposed by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy in the Ohio University, Athens, Ohio.**

Through each point of the straight line $x=my+h$ is drawn a chord of the parabola $y^2=4ax$, which is bisected in the point. Prove that this chord touches the parabola $(y-2ma)^2=8a(x-h)$.

8. **Proposed by ADOLPH BAILOFF, Durand, Wisconsin.**

If the two exterior angles at the base of a triangle are equal, the triangle is isosceles.

9. Proposed by J. C. GREGG, Brazil, Indiana.

Two circles intersect in A and B. Through A two lines CAE and DAF are drawn, each passing through a centre and terminated by the circumferences. Show that $CA\times AE=DA\times AF$. [*Euclid.*]

10. Proposed by ERIC DOOLITTLE, Instructor in Mathematics, State University of Iowa, Iowa City.

If MN be any plane, and A and B any point without the plane, to find a point P, in the plane, such that $AP+PB$ shall be a minimum.

11. Proposed by Miss LECTA MILLER, B. L., Professor of Natural Science and Art, Kidder Institute, Kidder Missouri.

A gentleman's residence is at the center of his circular farm containing $a=900$ acres. He gives to each of his $m=7$ children an equal circular farm as large as can be made within the original farm; and he retains as large a circular farm of which his residence is the center, as can be made after the distribution. Required the area of the farms made.

12. Proposed by J. F. W. SCHEFFER, A. M., Hagerstown, Maryland.

Let OA and OB represent two variable conjugate semi-diameters of the ellipse $\frac{y^2}{a^2}+\frac{y^2}{b^2}=1$. On the chord AB as a side describe an equilateral triangle ABC. Find the locus of C.

13. Proposed by HENRY HEATON, M. S., Atlantic, Iowa.

Through two given points to pass four spherical surfaces tangent to two given spheres.

14. Proposed by HENRY HEATON, M. S., Atlantic City, Iowa.

Through a given point to draw four circles tangent to two given circles.

15. Proposed by ISAAC L. BEVERAGE, Monterey, Virginia.

A man starts from the centre of a circular 10 acre field and walks due north a certain distance, then turns and walks south-west till he comes to the circumference, walking altogether 40 rods. How far did he walk before making the turn?

16. Proposed by H. C. WHITAKER, B. S., M. E., Professor of Mathematics, Manual Training School. Philadelphia, Pennsylvania.

Three lights of intensities 2, 4 and 5 are placed respectively at points the co-ordinates of which are (0,3) (4,5) and (9,0). Find a point in the plane of the lights equally illuminated by all of them.

CALCULUS.

Conducted by J. M. COLAW, Monterey, Va. All contributions to this department should be sent to him.

PROBLEMS.

1. Find the moment of inertia about the origin, of the area included within the parabola $y^2=4ax$, the line $x+y=4a$, and the axis of x.

[Selected from *Osborne's Differential and Integral Calculus.*]

2. **Proposed by E. S. LOOMIS, A. M., Ph. D., Professor of Mathematics, Baldwin University, Berea, Ohio.**

Show that the indeterminate form $\dfrac{x-\frac{2}{3}\sin x-\frac{1}{3}\tan x}{x^5}=\dfrac{-1}{20}$, when $x=0$.

[Ex. 51, p. 112, *Williamson's Differential Calculus.*]

3. **Proposed by H. C. WHITAKER, B. S., M. E., Professor of Mathematics, Manual Training School, Philadelphia, Pennsylvania.**

The product of two sides of a triangle is $6000(k^2)$, the length of the bisector of the included angle is 60 (b). What is the maximum area of the triangle, what is the greatest length of the third side. and what the total length swept over by the triangle, the bisector remaining fixed? [Selected from *Philadelphia Call*, 26 May. 1890.]

4. **Proposed by J. M. COLAW, Principal of High School, Monterey, Virginia.**

Three towns A, B, and C are in the same straight line. The distance from A to B is 20 miles and the distance from B to C is 80 miles. A pedestrian started from B for C and traveled at the variable rate of 10 miles an hour reciprocally as the cube root of his distance from A. In what time did he travel from B to C?

5. **Proposed by CHARLES E. MYERS, Canton, Ohio.**

The volume generated by the curve whose equation is $y^2=px$, revolving about its axis, is cut by a right cylinder whose equation is $y^2=px-x^2$, the axis of the latter passing through the focus of the former. Find the volume common to both by the formula $V=\int\int\int dxdydz$.

6. **Proposed by O. S. KIBLER, Superintendent of Schools, West Middleburg, Ohio.**

A string is wound spirally twenty times around a cylinder 20 feet high and 2 feet in diameter. Through what distance will a dove fly in unwinding the string keeping it tense at all times (1) flying in the same plane and (2) not flying in the same plane?

MECHANICS.

Conducted by B. F FINKEL, Kidder, Missouri. All contributions to this department should be sent to him.

PROBLEMS.

1. **Proposed by A. M. SCRIPTURE, A. M., Principal of Schools, New Hartford, New York.**

Suppose A and B to start at the same time from the same point in a level plane. A walks south, going 30 inches at a step, stepping twice in a second and beating a drum at the begining of each step. B steps west 30 inches every time he hears the drum beat. How far apart will they be at the end of 15 minutes, if the temperature of the air is $+41^\circ$ Fahrenheit?

2. **Proposed by WILLIAM SYMMONDS, A. M., Professor of Mathematics and Astronomy, Pacific College, Santa Rosa, California.**

The axis of a parabola coincides with a vertical line. What is the position of that focal chord through which a body would roll down in the least time?

3. **Proposed by CHARLES E. MYERS, Canton, Ohio.**

A spherical air-bubble, having risen from a depth of 1,500 feet in water, was one inch in diameter when it reached the surface; what was its diameter at the point of starting?

4. **Proposed by DeVOLSON WOOD, M.A.,C. E., Professor of Mechanical Engineering, Stevens Institute of Technology, Hoboken, New Jersey.**

A particle starts at rest and revolves in a circle with a uniform acceleration, acquiring a velocity v in t seconds. Find the locus of the foot of the perpendicular from the centre of the circle upon the resultant acceleration.

5. **Proposed by J. R. BALDWIN, A. M., Professor of Mathematics and Commercial Law, Davenport Business College, Davenport, Iowa.**

A 200 pound ball lies on a three legged table, having the legs equally distant apart and perpendicular to the plane of the top of the table. (1) What is the weight on each leg of the table not including the top when the ball is 2 feet, 3 feet, and 4 feet distant from the three legs? (2) If the ball is 2 feet, 3 feet, and 5 feet from the legs, what must be the weight of the top to keep from tipping and the weight on each leg excluding the top and also including the top?

DIOPHANTINE ANALYSIS.

Conducted by J. M. COLAW, Monterey, Va. All contributions to this department should be sent to him.

PROBLEMS.

1. **Proposed by EARL D. WEST, West Middleburg, Logan County, Ohio.**

It is required to divide a given square number into two such parts that each part will be a square number.

2. **Proposed by J. M. COLAW, Principal of High School, Monterey, Virginia.**

Find two numbers, such that the difference of their squares may be a cube, and the difference of their cubes a square.

3. **Proposed by O. S. KIBLER, Superintendent of Schools, West Middleburg, Logan County, Ohio.**

It is required to find three whole numbers in an arithmetical progression, such that the sum of every two of them shall be a square.

4. **Proposed by H. W. HOLYCROSS, Superintendent of Schools, Pottersburg, Union County, Ohio.**

What value of x will render $4x^4+12x^3-3x^2-2x+1$ a square?

AVERAGE AND PROBABILITY.

Conducted by B. F. FINKEL, Kidder, Missouri. All contributions to this department should be sent to him.

PROBLEMS.

1. **Proposed by Professor G. B. M. ZERR, A. M., Principal of High School, Staunton, Virginia.**

Three persons A, B, C, throw with three dice. They each stake $10.00 and the one who first throws at least ten with the three dice takes the whole stake. Find the expectation of each.

2. **Proposed by O. S. KIBLER, Superintendent of Schools, West Middleburg, Logan County, Ohio.**

What is the average area of a triangle formed by joining an angle of a square with any two points within the square?

3. **Proposed by MISS LECTA MILLER, B. L., Professor of Natural Science and Art, Kidder Institute, Kidder, Missouri.**

A deer, wounded at the corner of a square park, is equally liable to run in a straight line in any direction, from the corner of the park, and, at the same time, is also equally liable to drop dead before running a distance equal to the diagonal of the park. What is the chance that the deer will drop dead in the park?

MISCELLANEOUS.

Conducted by J. M. COLAW, Monterey, Va. All contributions to this department should be sent to him.

PROBLEMS.

1. **Proposed by Professor G. B. M. ZERR, A. M., Principal of High School, Staunton, Virginia.**

To divide the arc of a cycloid into eight equal parts.

2. **Proposed by SYLVESTER ROBINS, Long Branch Depot, New Jersey.**

Give the dimensions of thirteen rational trapezoids each one having 1885 for its parallel bisector; and as many more wherein each bisector is 1105.

3. **Proposed by J. A. CALDERHEAD, Limaville, Ohio.**

Given the simultaneous angular velocities of a body about the principal axes through its center of inertia, find the position of these axes in space at any assigned instant.

4. **Proposed by J K ELLWOOD, A. M., Principal of Colfax School, Pittsburg, Pennsylvania.**

I have two circular grindstones, each $\frac{1}{2}$ in. thick. One is 6 in. and the other $4\frac{1}{2}$ in. in diameter, the aperture at center of each being $1\frac{1}{2}$ in. If when in motion they are continually tangent to each other, and $\frac{1}{2}$ cu. in. is ground off the larger wheel and $\frac{1}{4}$ cu. in. off the smaller in the first hour, how must their speed be increased so that the same amount per hour may be ground off each wheel until one is worn out? If in the first hour the larger wheel makes a revolutions, and the smaller b, how many must each make in each succeeding hour?

QUERIES AND INFORMATION.

Conducted by J. M. COLAW, Monterey, Va. All contributions to this department should be sent to him.

The difinition of the root of an equation is that it must satisfy the equation if substituted in it, that is, that it must produce an identity. But in the equation

$\sqrt{x+4}-\sqrt{x-4}=4$, the value 5, which we get by solving, does not produce an identity. Please explain. L. B.

Answer: A square root, or any even root, calls for the double sign. The value 5 satisfies the equation if the second radical is taken with the negative sign, that is, if it assumes the form $\sqrt{x+4}+\sqrt{x-4}=4$. Under this more general aspect, viz.; that an even root has *two* signs, the value 5 is correct; as soon, however, as you restrict such radicals to one sign, viz.; that originally given, the value 5 is to be rejected, and the solution becomes impossible.

For instance, take $\sqrt{x+a}\pm\sqrt{x-a}=a$, where a is any whole number. Solving we get $x=\dfrac{a^2+4}{4}$, and substituting we get, $\dfrac{a+2}{2}\pm\dfrac{a-2}{2}=a$ or 2, according as we use + or − sign. From which we see that if $a=2$ either sign will give us a value of x that will satisfy the equation.

If $a>2$, the value of $x=a$ when the + sign is used will satisfy the equation, while if − sign is used the value of the algebraic sum equals 2 or a value less than a.

If $a<2$, the value $x=a$ will satisfy the equation for the − sign, while for the + sign the algebraic sum will be greater than a.

In the example, $a=4$, a value greater than 2, and we get $\dfrac{4+2}{2}-\dfrac{4-2}{2}=2$.

J. M. C.

"It is claimed that $\frac{1}{203}$ is the probability required in the following problem: There are 30 numbers, 1, 2, 3, 4, etc., in a box from which 6 numbers are drawn at random. What is the probability that the numbers 5, 7, and 12 will be included in the 6 numbers?

I would like to see a full explanation of it, if it is true.

W. L. Harvey, Portland, Maine.

Solution.—The number of combinations that can be formed from 30 numbers, taking 6 at a time, equals the number of ways six numbers can be drawn from 30 numbers. ∴ the number of ways 6 numbers can be drawn from 30 numbers $=\dfrac{30.\ 29.\ 28.\ 27.\ 26.\ 25}{1.\ 2.\ 3.\ 4.\ 5.\ 6}=n$

If the numbers 5, 7, and 12 are taken out their still remain 27 numbers. The number of ways that 6 numbers can be drawn from 30 numbers, including 5, 7, and 12, equals the number of ways 3 numbers can be drawn from 27 numbers, and making up the 6 numbers by taking 5, 7, and 12 with each set of 3 numbers drawn.

∴ The number of ways that 6 numbers can be drawn from 30 numbers, including 5, 7, and 12 $=\dfrac{27.\ 26.\ 25}{1.\ 2.\ 3.}=n'$. The required probability $=p=\dfrac{n'}{n}$.

$$\therefore\ p=\frac{27.\ 26.\ 25}{1.\ 2.\ 3}\times\frac{1.\ 2.\ 3.\ 4.\ 5.\ 6}{30.\ 29.\ 28.\ 27.\ 26.\ 25}=\frac{4.\ 5.\ 6}{30.\ 29.\ 28}=\frac{1}{203},\quad \text{or}$$

the odds are 202 to 1 against the event. J. M. C.

[In answer to Wm. E. May, Jonesboro, Tennessee, we shall endeavor to present in the several issues of the Monthly for 1894 matter of the kind that you suggest. You may be interested in Prof. Durell's article in this number.[J. M. C.